

## GENERALIZED FORCES IN THE DESCRIPTION OF CREEP PROCESSES IN BEAM ELEMENTS

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*The intensity of the creep process and the time to rupture in bending and twisting of beam elements with various shapes are estimated. These estimates are compared with the use of equivalent external generalized forces.*

**Key words:** creep, time to rupture, creep characteristics, generalized forces.

**Introduction.** Hypotheses used to construct the energy variant of the creep and long-strength theory were described in [1–3]: the process intensity is estimated by the power of energy dissipated during irreversible creep deformation  $W = \sigma_{ij}\eta_{ij}$  ( $\sigma_{ij}$  are the components of the stress tensor and  $\eta_{ij} = \dot{\varepsilon}_{ij}^c$  are the components of the creep strain rate tensor), and the measure of material damage is the value of energy dissipated during irreversible creep deformation  $A(t) = \int_0^t \sigma_{ij}\eta_{ij} dt$ . It was shown that the time to rupture  $t^*$  of a structurally stable material in the case of creep at a constant temperature is inversely proportional to the energy dissipation power  $W$  at the initial (steady) stage of creep:

$$Wt^* = \text{const.} \quad (1)$$

In the general case, the creep process depends substantially on its duration. The energy variant usually offers a satisfactory description of moderate-duration processes accompanied by viscous rupture (the process duration is normally tens or hundreds of hours).

The validity of Eq. (1) was checked experimentally for several structural alloys in uniaxial and plane stress states. In the case of spatial loading, the energy dissipation power was determined by the formula

$$W_0 = \sigma_e \eta_e,$$

where  $\sigma_e$  and  $\eta_e$  are the equivalent stress and strain rate of creep. By using the Mises criterion and the associated flow law for isotropic materials, we can replace the equivalent stress and strain rate by the intensity of the corresponding tensors:

$$\sigma_i = ((3/2)\bar{\sigma}_{ij}\bar{\sigma}_{ij})^{1/2}, \quad \eta_i = ((2/3)\bar{\eta}_{ij}\bar{\eta}_{ij})^{1/2}.$$

Here,  $\bar{\sigma}_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0$  are the components of the stress tensor deviator,  $\sigma_0 = \delta_{ij}\sigma_{ij}/3$ , and  $\bar{\eta}_{ij} = \eta_{ij}$  are the components of the strain rate tensor (under the condition of material incompressibility). Within the framework of the energy approach, the quantities  $\sigma_e$  and  $\eta_e$  can be considered as the generalized stresses and generalized strain rates of creep, respectively, and their product can be considered as the power of energy dissipation in a unit volume of the body. If a uniform stress–strain state arises in the body under the action of external loads, the energy dissipation power  $W$  and the amount of accumulated damages  $A(t)$  in each elementary volume of the body are equal to the average values over the body volume. Therefore, if the energy losses of nonmechanical nature are ignored, the corresponding powers of internal ( $W = \sigma_e \eta_e$ ) and external ( $W = Q\dot{q}$ ) generalized forces are equal to each other.

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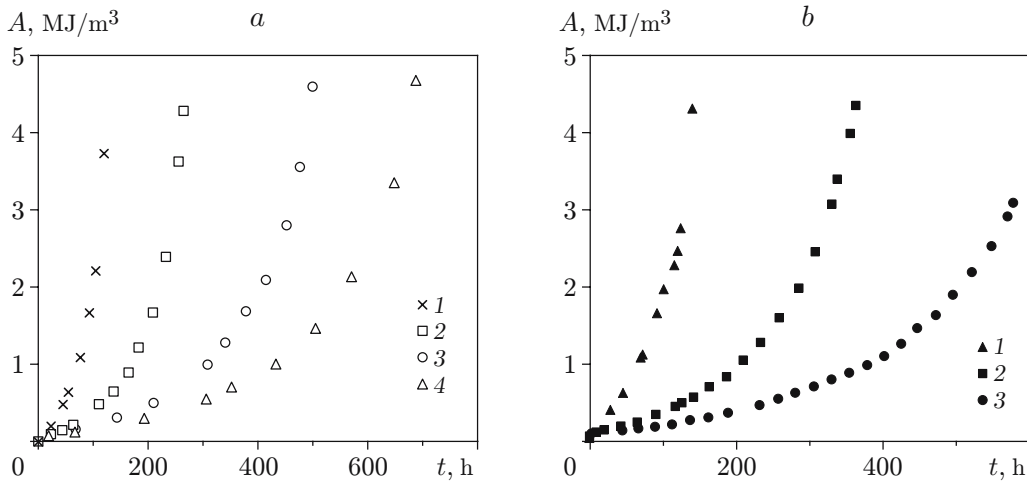


Fig. 1. Creep diagrams under twisting for the D16T alloy at  $T = 250^\circ\text{C}$  and a constant twisting moment: (a) thin-walled samples with  $M = 51.24$  (1) 44.92 (2), 41.14 (3), and 38.36 N·m (4); (b) solid samples with  $M = 92.57$  (1), 77.16 (2), and 69.34 N·m (3).

In practice, external loads can be replaced by certain generalized forces  $Q$  and external displacements  $q$  or their rates  $\dot{q}$ . It seems reasonable to use these quantities to estimate the time to rupture of various structural elements.

Figure 1 shows the creep diagrams obtained in experiments with twisting of thin-walled and solid cylindrical samples made of the D16T aluminum alloy at a temperature  $T = 250^\circ\text{C}$  and a constant twisting moment  $M$ . The outer diameter of the samples was  $D = 20$  mm, and the wall thickness of the tubular samples was 1 mm.

The value of energy dissipated in a unit volume of the body  $A = M\varphi(t)/V$  ( $V = Sl$  is the working volume and  $S$  is the cross-sectional area of the sample) was calculated during the experiment over a given working length  $l$  on the basis of the twisting angle  $\varphi(t)$ . Using the initial parts of the diagrams  $A(t)$ , we calculated the specific power of energy dissipation  $W$ . The product of this quantity and the time to rupture  $t^*$  had the same average value  $Wt^* \approx 1$  MJ/m³ for both thin-walled and solid samples [4].

Figure 2a shows the results of experiments obtained for tubular samples (outer diameter  $D = 20$  mm and inner diameter  $d = 18$  mm) made of the D16T alloy at a temperature  $T = 250^\circ\text{C}$  and different combinations of tension (compression) with twisting, which are indicated in Fig. 2b and ensure an identical specific power of energy dissipation  $W \approx 2.7 \cdot 10^{-3}$  MJ/(m³·h) [2] ( $\sigma$  and  $\tau$  are the axial and tangential stresses, respectively). The specific dissipated energy of the body was calculated by the formula

$$A(t) = \frac{1}{V} (P \Delta l(t) + M\varphi(t)),$$

where  $P$  is the axial load and  $\Delta l(t)$  is the sample elongation. In experiments illustrated in Figs. 1 and 2, the product was  $Wt^* \approx 1$  MJ/m³. It should be noted that the intensity of the creep process under shear for the D16T alloy is much greater than the intensity of the creep process under tension, if the measure of equivalence is taken to be  $\sigma_i$  (the dashed curve in Fig. 2b corresponds to  $\sigma_i = \text{const}$ ). Nevertheless, Eq. (1) is experimentally validated for both the uniform and nonuniform stress-strain states of a nonisotropic medium.

The above-presented results of three series of experiments confirm the consistency of the assumption that it is possible to compare the intensity of the process of high-temperature creep and time to rupture of single-type structural elements in terms of the volume-averaged powers of energy dissipation  $W$ . Yet, there arises a question whether it is possible to compare the strain-strength behavior of structural elements subjected to various external actions. Results of three series of experiments on high-temperature deformation of beam elements made of an isotropic material (St. 45 steel) at a temperature  $T = 725^\circ\text{C}$  are given below: tension-compression of cylindrical rods, bending of beams with different profiles, and twisting of solid and thin-walled rods. The initial creep curves for St. 45 steel samples can be found in [5].

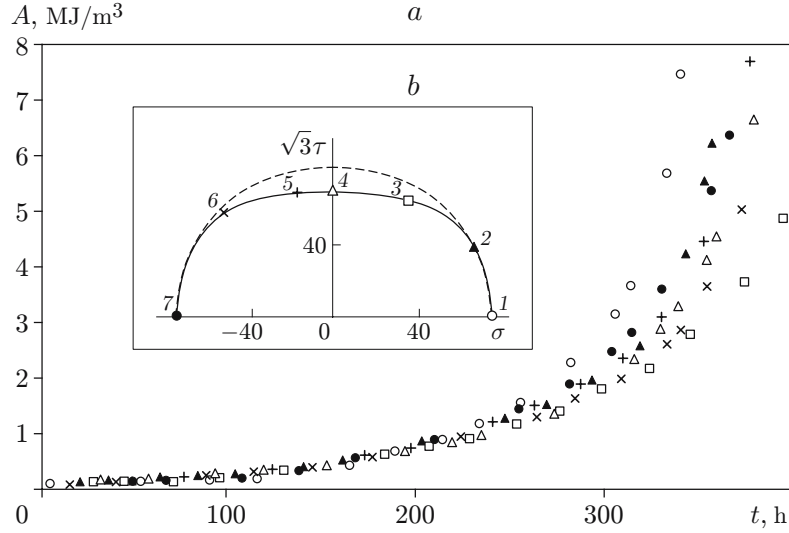


Fig. 2. Creep diagrams for thin-walled samples (a) for different combinations of tension (compression) and twisting (b) for the D16T alloy at  $T = 250^\circ\text{C}$ : 1)  $\sigma = 78.48$  MPa and  $t^* = 340$  h; 2)  $\sigma = 69.65$  MPa,  $\tau = 20.6$  MPa, and  $t^* = 360$  h; 3)  $\sigma = 36.3$  MPa,  $\tau = 36.3$  MPa, and  $t^* = 410$  h; 4)  $\tau = 37.96$  MPa and  $t^* = 380$  h (4); 5)  $\sigma = -16.68$  MPa,  $\tau = 37.77$  MPa, and  $t^* = 375$  h; 6)  $\sigma = -54.94$  MPa,  $\tau = 31.88$  MPa, and  $t^* = 375$  h; 7)  $\sigma = -78.48$  MPa and  $t^* = 365$  h.

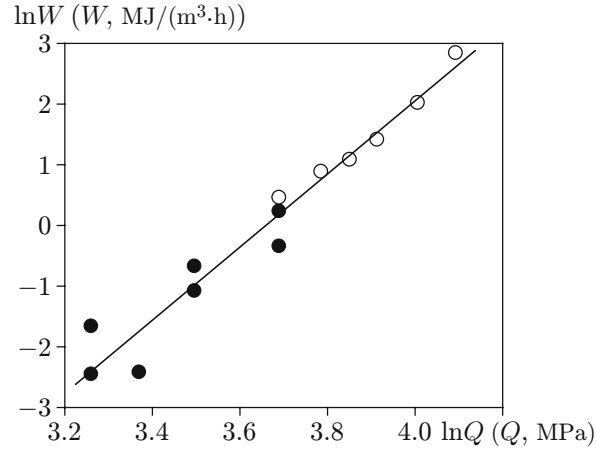


Fig. 3. Intensity of the creep process versus the generalized force under tension (open points) and compression (filled points) for St. 45 steel at  $T = 725^\circ\text{C}$ : the solid curve approximates the dependence  $W = B\sigma^n$  at  $n = 6$  and  $B = 3 \cdot 10^{-10} \text{ MPa}^{1-n} \cdot \text{h}^{-1}$ .

**Comparative Estimates of Deformation of Beam Elements under Tension and Bending.** Figure 3 shows the results of experiments on creep of samples under tension and compression at the steady stage of the process. In the case of tension (under the condition of material incompressibility  $V = S_0 l_0 = Sl$ ), we have

$$dA = \frac{1}{V} P dl = \frac{P}{S} \frac{dl}{l} = Q_1 dq_1,$$

where  $Q_1 = P/S = \sigma$  is the generalized force and  $dq_1 = d(\ln(l/l_0))$  is the generalized displacement. Approximating the creep process at the steady stage by the dependence

$$W = BQ^n, \quad (2)$$

we obtain  $n = 6$  and  $B_1 = 3 \cdot 10^{-10} \text{ MPa}^{1-n}/\text{h}$ .

Figure 4a shows the results of experiments on beam bending under creep conditions at the steady stage. In the case of bending under the action of a constant bending moment, we have

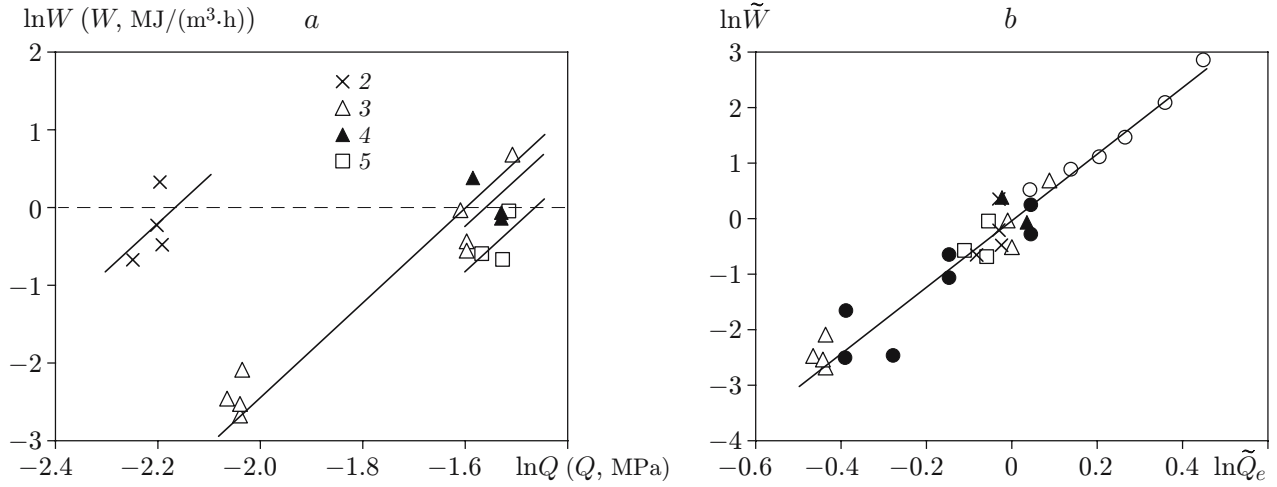


Fig. 4. Intensity of the creep process versus the generalized force  $\ln W \sim \ln Q$  (a) and  $\ln \tilde{W} \sim \ln \tilde{Q}_e$  (b) in the process of bending of beams made of St. 45 steel ( $T = 725^\circ\text{C}$ ): the solid curves approximate the dependence  $W = B\sigma^n$  at  $n = 6$  and the mean value of  $B$ ; the dashed curve is the reference value  $\tilde{W}_0 = 1 \text{ MJ}/(\text{m}^3 \cdot \text{h})$ ; beam with  $h = 10 \text{ mm}$  and  $b = 20 \text{ mm}$  (2), beam with  $h = 20 \text{ mm}$  and  $b = 10 \text{ mm}$  (3), T-beam with a compressed peak (4), and T-beam with a stretched peak (5).

TABLE 1

Characteristics of the Creep Process of Beam Elements  
Made of St. 45 Steel at  $T = 725^\circ\text{C}$  and Equivalence Coefficients  $\lambda$ ,  $\lambda^{\text{kin}}$ , and  $\lambda^{\text{st}}$

$k$	Type of loading	$B$ , $\text{MPa}^{1-n} \cdot \text{m}^{-n} \cdot \text{h}^{-1}$	$Q_0$ , $\text{MPa} \cdot \text{m}$	$\lambda$ , $\text{m}^{-1}$	$\lambda^{\text{kin}}$ , $\text{m}^{-1}$	$\lambda^{\text{st}}$ , $\text{m}^{-1}$
2	Bending	$4.46 \cdot 10^5$	0.082	335.0	123.8	406.0
3		$1.48 \cdot 10^4$	0.145	190.0	61.1	200.2
4		$1.19 \cdot 10^4$	0.150	183.0	57.9	192.5
5		$6.65 \cdot 10^3$	0.165	166.0	57.8	188.7
6	Twisting	$9.18 \cdot 10^4$	0.107	259.5	221.9	259.8
7		$7.88 \cdot 10^3$	0.161	172.4	157.7	173.2

**Note.** The value  $k = 1$  corresponds to experiments on tension–compression of cylindrical samples.

$$dA = \frac{1}{V} M d\varphi = \frac{M}{S} \frac{d\varphi}{l_0} = Q dq,$$

where  $Q = M/S$  [ $\text{N} \cdot \text{m}/\text{m}^2$ ] is the generalized force in bending of beams with four different profiles and  $dq = d\varphi/l_0 = d\alpha$  is the generalized displacement. In our experiments, the curvature  $\alpha$  was determined by the beam deflection  $\Delta(t)$  at the middle point over the base length  $l_0 = 100 \text{ mm}$  by the formula  $\alpha = 8\Delta/l_0^2$ . Using the same approximation (2) at the stage of steady creep as in the tension–compression experiments, we obtain  $n = 6$  and the values of  $B$  listed in Table 1 for bent beam elements of four types (see Fig. 4): 1) beam with a height  $h = 10 \text{ mm}$  and width  $b = 20 \text{ mm}$  (points 2 in Fig. 4); 2) beam with a height  $h = 20 \text{ mm}$  and width  $b = 10 \text{ mm}$  (points 3 in Fig. 4); 3) T-beam with a compressed peak (points 4 in Fig. 4); 4) T-beam with a stretched peak (points 5 in Fig. 4). The height of the T-beams was  $h = 20 \text{ mm}$ , the height of the peak was  $h_1 = 4\text{--}6 \text{ mm}$ , the width of the peak was  $b_1 = 20 \text{ mm}$ , and the width of the peak protruding was  $b_2 = 6 \text{ mm}$ .

In the case of tension and bending of beam elements made of this material and having different profiles, Eq. (2) yields identical values of the power index  $n$ , but different values of the coefficient  $B$  and generalized force  $Q$ . Comparisons of the processes in terms of the external generalized forces  $Q$  (even for one-dimensional cases

of loading) are invalid: the generalized forces in tension and bending have different dimensions and exert different actions on the body. These generalized forces can be converted to a certain equivalent generalized force  $Q_e$ , using two approaches: 1) normalization to the quantity  $Q_0$ , which ensures the same specific power of energy dissipation for each element, i.e., conversion of all generalized forces  $Q$  corresponding to different types of experiments to a dimensionless generalized force  $\tilde{Q}_e = Q/Q_0$ ; 2) introduction of equivalence coefficients  $\lambda$ , which convert the generalized forces for different types of loading to one equivalent quantity  $Q_e$  characterizing the behavior of some beam element taken as a reference element.

The first approach is based on the assumption that it is possible to compare the creep processes of arbitrary beam elements in terms of their specific powers of energy dissipation. In the considered range of powers, we choose some value  $W_0$  as a reference value, for instance,  $W_0 = 1 \text{ MJ}/(\text{m}^3 \cdot \text{h})$  ( $\ln W = \ln W_0$  is the dashed curve in Fig. 4a). Relation (2) is valid for all beam elements (for the reference process with the power  $W_0$ , the values of  $Q_0$  are listed in Table 1 for all types of beams considered). Normalizing the values of  $W$  and  $Q$  to the corresponding values for the reference process, we obtain a dimensionless (normalized) relation, which is identical for all types of beam elements:

$$\tilde{W} = \tilde{Q}_e^n, \quad \tilde{W} = W/W_0, \quad \tilde{Q}_e = Q/Q_0. \quad (3)$$

The data plotted in Figs. 3 and 4a are presented in the dimensionless form in Fig. 4b, where identical dimensionless values of  $\tilde{Q}_e$  correspond to one value of  $\tilde{W}$ .

Equation (1) yields

$$\tilde{Q}_e^n t^* = t_0^*. \quad (4)$$

Thus, replacing the characteristic  $B$  by a new characteristic  $Q_0$ , which takes into account the properties of the material and the specific properties of the beam element geometry, makes it possible to compare the intensities of the processes; by using Eq. (4), it is also possible to compare the times to rupture.

In accordance with the second approach, we have

$$W = B_k Q_k^n = B Q_e^n, \quad (5)$$

where  $Q_e = \lambda_1 Q_1 = \lambda_k Q_k$  ( $k = 2, 3, 4, 5$ ). If we use the tension experiment as a reference, we obtain  $W = B_1 Q_1^n = B(\lambda_1 Q_1)^n = (B\lambda_1^n)Q_1^n$ . In this case, we can assume that  $\lambda_1 = 1$  and  $B = B_1$ . As a result, we obtain

$$W = B_1 Q_1^n = B_1 Q_e^n = B_1 (\lambda_k Q_k)^n \Rightarrow \lambda_k = Q_1/Q_k = Q_{01}/Q_{0k}; \quad (6)$$

$$W_0 = B_1 Q_{01}^n = B_k Q_{0k}^n \Rightarrow Q_{01}/Q_{0k} = \lambda_k = (B_k/B_1)^{1/n}. \quad (7)$$

The equivalence coefficients  $\lambda$  obtained by Eq. (7) are listed in Table 1.

The coefficients  $\lambda$  should take into account the specific features of the action of the external generalized force  $Q$  on the body, the beam element geometry, and the distribution of internal stresses. It is reasonable to obtain the approximate values of these coefficients by using only the data of experiments on the creep of the beam element under tension, which corresponds to the characteristics  $B_1$  and  $n$ . These approaches known in technical publications are based on considering statically admissible fields of stresses and kinematically possible fields of strain rates.

Let us consider a statically admissible field of stresses in a bent beam in the form of a "limiting" state, where the stresses in the stretched and compressed regions are identical and independent of the distance to the neutral plane:  $|\sigma^+| = |\sigma^-| = \text{const}$ . Then, we have

$$M = \int_S \sigma y dS = |\sigma| \int_S |y| dS.$$

Introducing the notation  $J_{\text{st}} = \int_S |y| dS$ , we obtain  $Q_e = |\sigma| = M/J_{\text{st}}$ . Then, Eq. (6) yields

$$W = B_1 Q_e^n = B_1 (\lambda^{\text{st}} Q)^n,$$

where  $Q = M/S$  and  $Q_e = M/J_{\text{st}} = SM/(J_{\text{st}}S)$ . Therefore, we have  $\lambda^{\text{st}} = S/J_{\text{st}}$ .

Let us consider a kinematically possible field of strain rates. Let the strain rates  $\eta$  be defined in the form of linear functions of the coordinate over the beam height  $\eta = \dot{\alpha}y$ . It follows from the equalities  $W = \sigma\eta = B_1\sigma^n$

that  $\eta = B_1\sigma^{n-1}$  and  $\sigma = (\eta/B_1)^{1/(n-1)}$ . Using these relations, we can write the expression  $M = \int_S |\sigma||y| dS$  at the steady stage ( $\dot{\alpha} = \text{const}$ ) as

$$M = \left(\frac{\dot{\alpha}}{B_1}\right)^{1/(n-1)} \int_S |y|^{n/(n-1)} dS,$$

whence it follows that  $\dot{\alpha} = M^{n-1} B_1 / J_{\text{kin}}^{n-1}$ , where  $J_{\text{kin}} = \int_S |y|^{n/(n-1)} dS$ . Multiplying  $\dot{\alpha}$  by  $Q = M/S$ , we obtain

$$W = Q\dot{q} = B_1 \left[ \left(\frac{S}{J_{\text{kin}}}\right)^{(n-1)/n} \frac{M}{S} \right]^n = B_1 (\lambda^{\text{kin}} Q)^n,$$

where  $\lambda^{\text{kin}} = (S/J_{\text{kin}})^{(n-1)/n}$ .

In accordance with [6], at a fixed external load, the energy dissipation power  $W_{\text{st}}$  should be greater than the true values of  $W$  at statically admissible fields of stresses and  $W_{\text{kin}}$  should be smaller than the true value at kinematically possible fields of strain rates. Table 1 gives the mean values of the upper and lower estimates of the equivalence coefficients for the above-considered beam profiles ( $\lambda^{\text{kin}} \leq \lambda \leq \lambda^{\text{st}}$ ). Thus, we found an interval containing the true value of  $\lambda$ , which can be used in practice.

The value of  $J_{\text{kin}}$  depends on the creep index  $n$  and, as  $n$  increases, tends to the value  $J_{\text{st}}$ , which is independent of  $n$ , i.e., the difference between  $\lambda^{\text{st}}$  and  $\lambda^{\text{kin}}$  decreases with increasing  $n$ .

Most technical publications with calculations of beam bending in the case of creep involve the hypothesis of plane sections, which implies that the strain rate is distributed over the beam height in accordance with the linear law  $\eta = \dot{\alpha}y$ . The hypothesis of the ‘‘limiting’’ stress state ( $|\sigma^+| = |\sigma^-| = \text{const}$ ) is used much more seldom. The results of numerical calculations and comparisons with experimental data (see Table 1) for high-temperature creep processes, however, show that the hypothesis of the ‘‘limiting’’ stress state is preferable. Therefore, the use of the specific power of energy dissipation offers a satisfactory description of the intensities of the high-temperature creep processes in structural elements and allows their comparisons.

**Comparative Estimates of Deformation of Beam Elements under Tension and Twisting.** As we made above, we compare the intensities of beam deformation processes under tension and twisting. In the case of beam twisting, we have

$$dA = \frac{1}{V} M d\varphi = \frac{M_t}{S} \frac{d\varphi}{l} = Q dq,$$

where  $Q = M/S$  [ $\text{N} \cdot \text{m}/\text{m}^2$ ] is the generalized force under twisting and  $dq = d\varphi/l$  is the generalized displacement. In the case of a uniform stress–strain state, which is formed in the case of twisting of a thin-walled tube with outer and inner radii  $R$  and  $r = \beta R$ , respectively, the relation between the generalized force  $Q$  and shear stress  $\tau$  is established via the external moment:

$$Q = \frac{M_t}{S} = \frac{1}{S} \int_0^{2\pi} \int_{\beta R}^R \tau \rho^2 d\rho d\varphi = \tau \frac{2R(1 + \beta + \beta^2)}{3(1 + \beta^2)}.$$

As  $\beta \rightarrow 1$ , we have  $Q = \tau R$ , i.e., as in the case of bending, the dimension of the generalized force under twisting differs from the dimension of the generalized force under tension.

Figure 5a shows the results of experiments on twisting of thin-walled tubes with  $R = 10$  mm and  $r = 9$  mm (points 7) and solid cylindrical beams with  $D = 20$  mm (points 6) at the steady stage of the creep process. Processing the experimental data for the steady stage with the use of the power-law dependence (2), we obtain the values of the coefficient  $B$  at  $n = 6$  for thin-walled cylinders and solid samples (see Table 1).

As in the case of beam bending, we consider two approaches for conversion of the generalized forces  $Q$  to an equivalent quantity  $Q_e$ . In accordance with the first approach, we take the same reference value  $W_0 = 1$  MJ/( $\text{m}^3 \cdot \text{h}$ ) ( $\ln W = \ln W_0$  is shown by the dashed curve in Fig. 5a) and convert the equation to the dimensionless (normalized) form (3). The corresponding values of  $Q_0$  are listed in Table 1. The dependence  $\ln \tilde{W} \sim \ln \tilde{Q}_e$  and also the data of tension experiments (see Fig. 3) are shown in Fig. 5b. Thus, we have the same dependence  $\tilde{W}(\tilde{Q}_e)$  under loading by the tensile (compressive) load and by the bending and twisting moments.

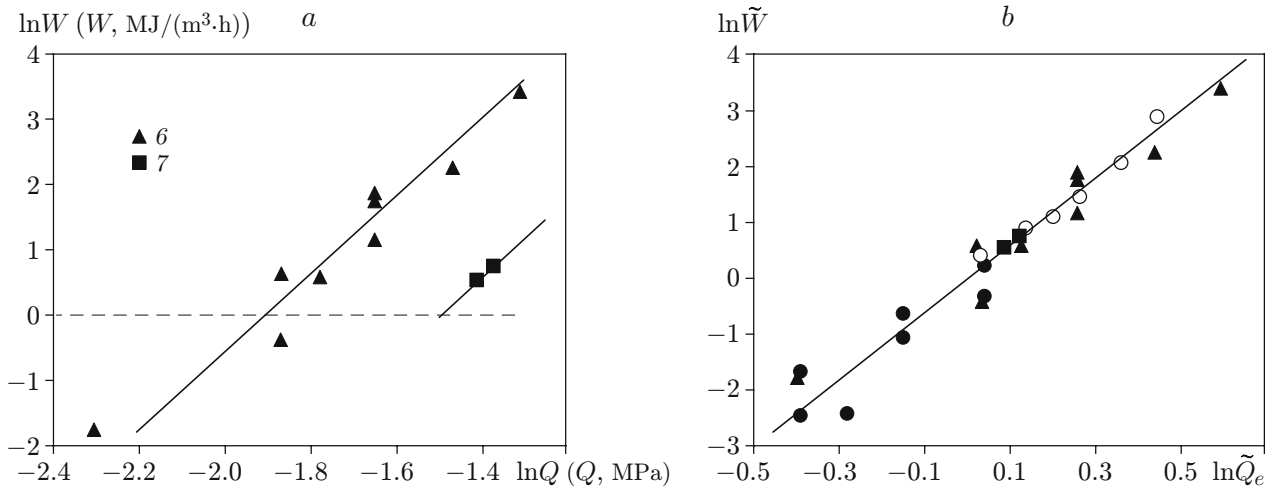


Fig. 5. Intensities of the creep process versus the generalized force  $\ln W \sim \ln Q$  (a) and  $\ln \tilde{W} \sim \ln \tilde{Q}_e$  (b) in the case of twisting of solid beams (6) and thin-walled cylinders (7) made of St. 45 steel ( $T = 725^\circ\text{C}$ ): the solid curves approximate the dependence  $W = B\sigma^n$  at  $n = 6$  and the mean value of  $B$ ; the dashed curve is the reference value  $W_0 = 1 \text{ MJ}/(\text{m}^3 \cdot \text{h})$ .

In accordance with the second approach, the creep process under tension is the reference process:  $Q_e = \lambda_k Q_k$ . Here, the equivalence coefficients in the case of twisting are determined from Eq. (7) (see Table 1).

It follows from Eq. (6) that  $W = B_1 Q_e^n = B_1 (\lambda_7 Q_7)^n$ , where  $Q_e = \sigma$  under uniaxial tension and  $Q_7 = R\tau$  under twisting of a thin-walled tube in the case of uniform stress-strain states. Using the equality of the stress intensities  $\sigma = \sqrt{3}\tau$  as the equivalence criterion, we have  $\lambda_7^{\text{st}} = \sqrt{3}/R$ . For  $R = 10 \text{ mm}$ , we obtain the value  $\lambda_7^{\text{st}} = 173.2 \text{ m}^{-1}$ , which is close to the equivalence coefficient obtained directly in experiments.

Using statically admissible fields of stresses and kinematically possible fields of strain rates, we can find the approximate values of the equivalence coefficient  $\lambda_6$ . Thus, for a statically possible stress state  $\tau = \text{const}$  over the beam cross section, we have  $Q_6 = M/S = (2/3)R\tau$ . Using the equivalence criterion  $\tau = \sigma/\sqrt{3}$  and Eq. (7), we obtain  $\lambda_6^{\text{st}} = Q_1/Q_6 = (3\sqrt{3})/(2R) = 259.8 \text{ m}^{-1}$ .

For a kinematically possible field of strain rates, for instance, for the field  $\dot{\gamma} = \rho\dot{\phi}/l$ , we use the equivalence criterion and obtain  $W = \dot{\gamma}\tau = B_1\sigma^n = B_1(\sqrt{3}\tau)^n$ . Substituting the value

$$\tau = \left( \frac{\rho\dot{\phi}}{B_1 l (\sqrt{3})^n} \right)^{1/(n-1)}$$

into the expression

$$M = \int_S \tau \rho dS,$$

we obtain

$$\left( \frac{M}{S} \right)^n \left( \frac{S}{J_{\text{kin}}} \right)^{n-1} = \left( \frac{\sigma}{\sqrt{3}} \right)^n.$$

In accordance with Eq. (6), we have  $\lambda_6^{\text{kin}} = \sqrt{3}(S/J_{\text{kin}})^{(n-1)/n}$ . For twisting of a solid beam with  $R = 10 \text{ mm}$  at  $n = 6$ , we find  $\lambda_6^{\text{kin}} = 221.9 \text{ m}^{-1}$ . In this case, we have  $\lambda^{\text{kin}} \leq \lambda \leq \lambda^{\text{st}}$ , as in the case of beam bending; the difference between  $\lambda^{\text{kin}}$  and  $\lambda^{\text{st}}$  decreases with increasing  $n$ . It should also be noted that the results of calculations under the assumption of the limiting state  $\tau = \text{const}$  over the section radius (statically admissible field of stresses) are in better agreement with experimental data than the results calculated under the assumption of the linear distribution of the shear rates over the radius.

Using the experimental data presented, it is possible to compare the strain-strength behavior of structural elements in terms of the equivalent external generalized forces  $Q_e = \lambda_k Q_k$ . Figure 6 shows the creep diagrams of various beam elements made of St. 45 steel at  $T = 725^\circ\text{C}$  and  $Q_e = 44 \text{ MPa}$ . It is seen that close values of intensity

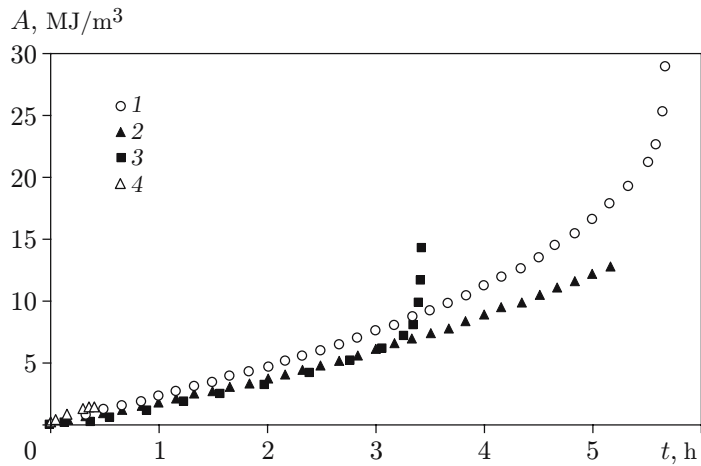


Fig. 6. Creep diagrams for beam elements at  $Q_e = 44$  MPa for St. 45 steel at  $T = 725^\circ\text{C}$ : tension of a cylindrical sample (1), twisting of a solid cylindrical sample (2), twisting of a thin-walled cylindrical sample (3), and bending of a rectangular beam (4).

and duration of the processes correspond to close values of the generalized forces. The shorter duration of the creep process in the case of twisting of a thin-walled sample, as compared with the duration of the creep process under tension, is explained by buckling of the thin-walled structure. Bending experiments were performed only for low values of strains.

**Conclusions.** The intensity of the creep processes and the time to rupture of beam elements made of the same material at a fixed temperature can be compared in terms of the volume-averaged powers of energy dissipation calculated with the use of external generalized forces converted to an equivalent quantity.

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